Recent evolutions of DSST propagator in Orekit library

Bryan Cazabonne

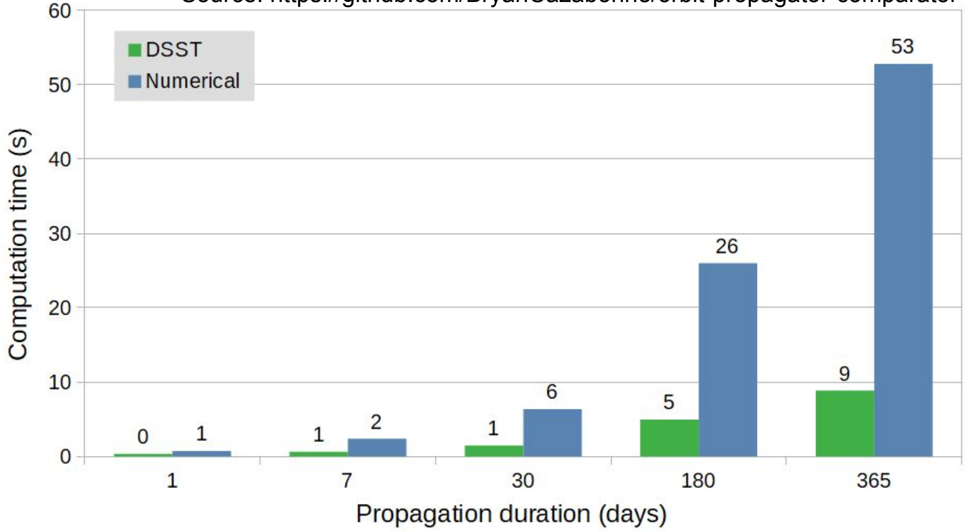
Orekit Talk - 23th of May 2023

Introduction

Draper Semi-analytical Satellite Theory (DSST)

- **Accuracy** of numerical propagation
- **Characteristic speed** of analytical propagation

Source: https://github.com/BryanCazabonne/orbit-propagator-comparator



Computation time comparison for a GEO test case

- ~ 30 meters difference in sma after 30 days

DSST development in Orekit

Orekit 6.0 | 2013

First release of DSST propagator in Orekit.

...

Orekit 10.0 | 2019

First release of DSST least squares orbit determination

Orekit 10.3 | 2020

Refined validation of DSST State Transition Matrix

Orekit 11.1 | 2022

Implementation of the Extended Semi-analytical Kalman Filter (ESKF)

Orekit 11.3 | 2022

First version of the Unscented Semi-analytical Kalman Filter

The Semi-analytical Batch Least Square Orbit Determination

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The State Transition Matrix

A semi-analytical formulation for the partial derivatives

$$\Phi_{t,t_0} = \frac{\partial \boldsymbol{Y}_t}{\partial \underline{\boldsymbol{a}}^*(t)} \quad . \quad G$$

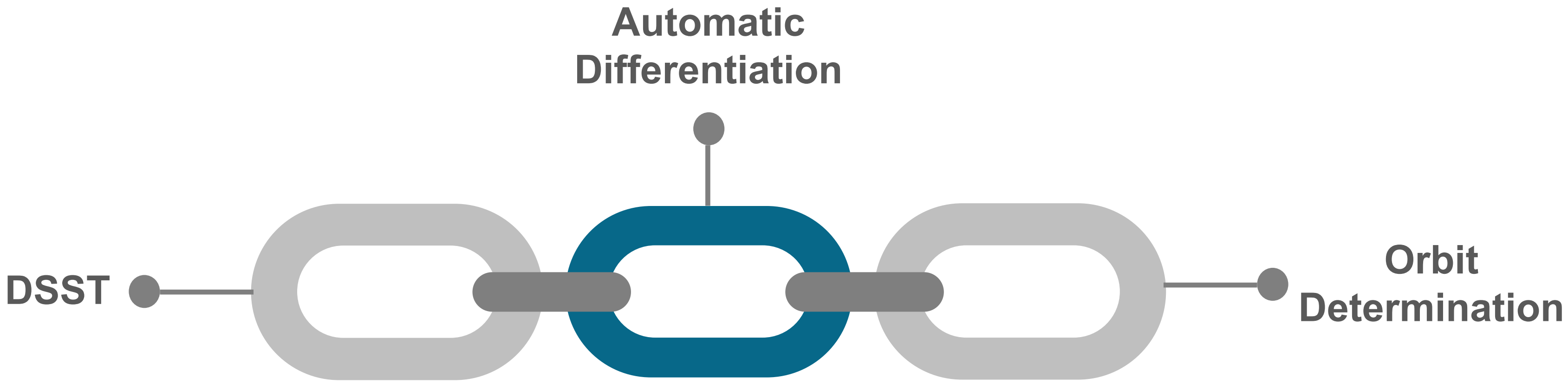
The Green matrix (Green, 1979)

$$G = [I_6 + B_1][B_2 \quad B_3] + [0_{6,6} \quad B_4]$$

$$B_1 = \left[\frac{\partial \boldsymbol{\varepsilon\eta}(\overline{\boldsymbol{a}})}{\partial \overline{\boldsymbol{a}}(t)}\right]_{6\times6} \quad B_2 = \left[\frac{\partial \overline{\boldsymbol{a}}(t)}{\partial \overline{\boldsymbol{a}}_0}\right]_{6\times6} \quad B_3 = \left[\frac{\partial \overline{\boldsymbol{a}}(t)}{\partial \underline{\boldsymbol{c}}}\right]_{6\times(s-6)} \quad B_4 = \left[\frac{\partial \boldsymbol{\varepsilon\eta}(\overline{\boldsymbol{a}})}{\partial \underline{\boldsymbol{c}}}\right]_{6\times(s-6)}$$

The magic of Orekit

Target



The Automatic Differentiation

Goal

$$\left[Y_i \quad \frac{\partial Y_i}{\partial Y_1} \quad \frac{\partial Y_i}{\partial Y_2} \quad \cdots \quad \frac{\partial Y_i}{\partial Y_6} \quad \frac{\partial Y_i}{\partial \underline{c}_1} \quad \cdots \quad \frac{\partial Y_i}{\partial \underline{c}_N}\right]$$

```
double f(double x, double y) {
    if (x > 0) {
        return x + g(y);
    } else {
        return x - g(y);
    }
}
```

↓

```
T f(T x, T y) {
    if (x.getReal() > 0) {
        return x.add(g(y));
    } else {
        return x.subtract(g(y));
    }
}
```

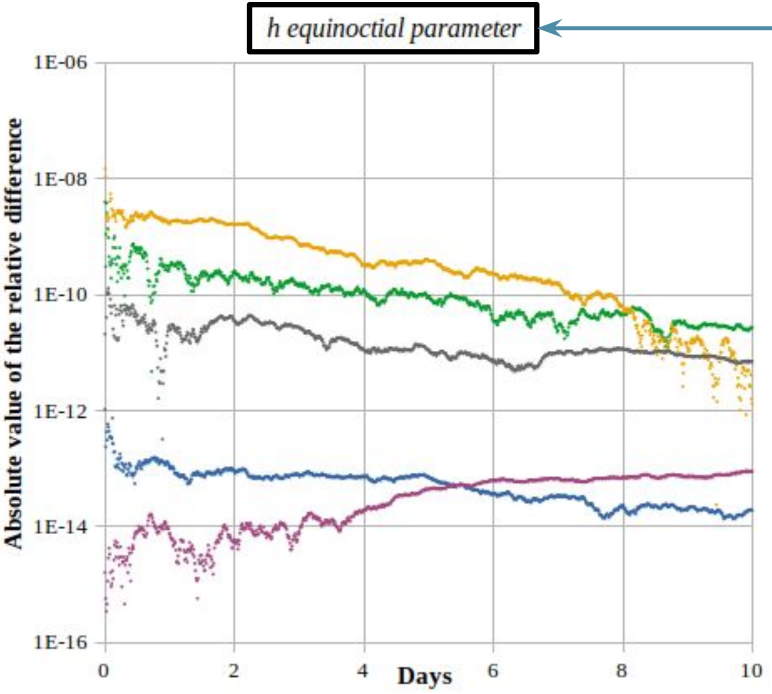
Gain

- **Safer** implementation
- **Simpler** validation

Validation

- Testing B_1 and B_2 computed with automatic differentiation via a comparison with a finite differences calculation → J_2 only test case with a 10-day forward propagation

Validation of the State Transition Matrix

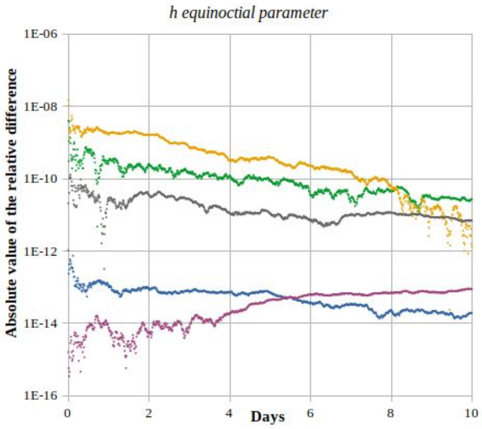


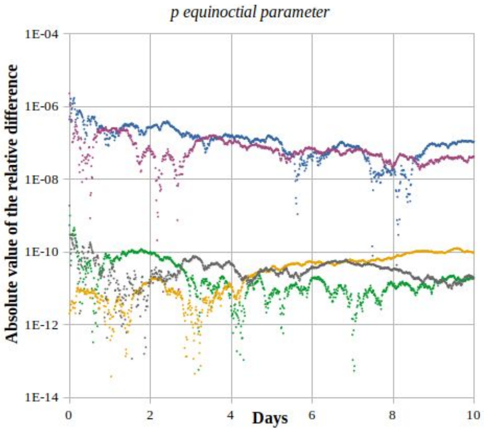
Partial derivatives of h **mean** equinoctial parameter

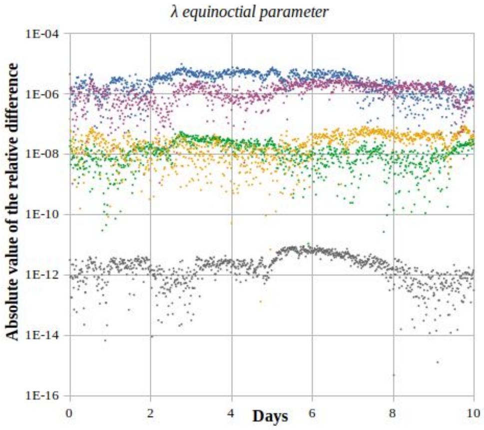
- with respect to $\bar{a}_0$
- with respect to $\bar{h}_0$
- with respect to $\bar{k}_0$
- with respect to $\bar{p}_0$
- with respect to $\bar{q}_0$

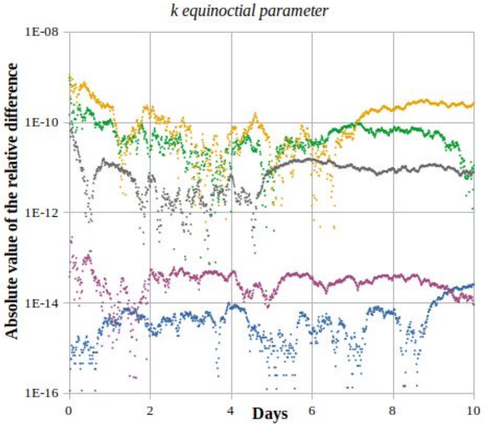
$$B_2 = \left[\frac{\partial \overline{\boldsymbol{a}}(t)}{\partial \overline{\boldsymbol{a}}_0}\right]_{6\times 6}$$

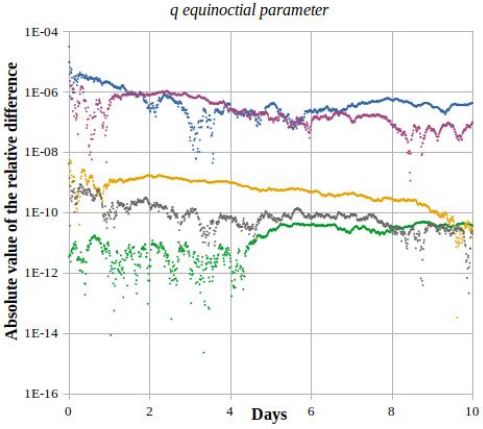
Validation of the State Transition Matrix





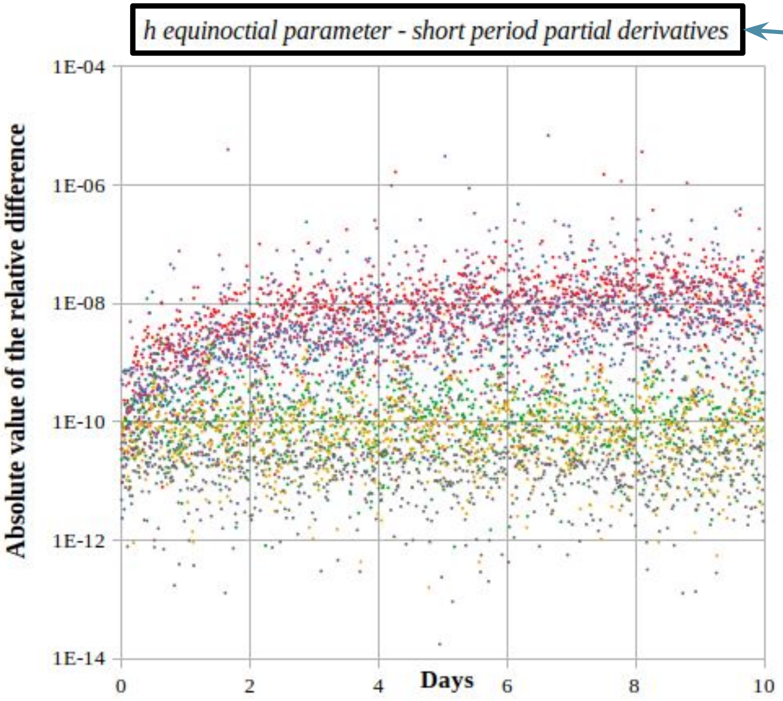






(Cazabonne and Cefola, AAS Paper 21-309)

Validation of the State Transition Matrix

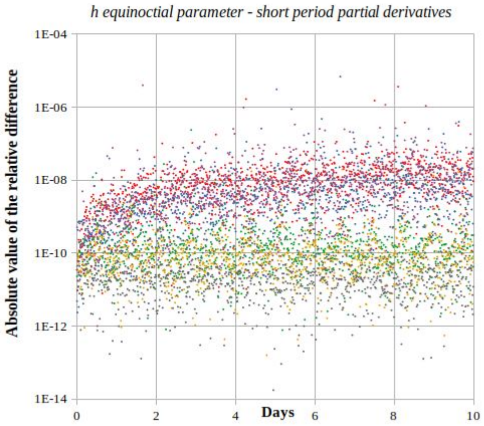


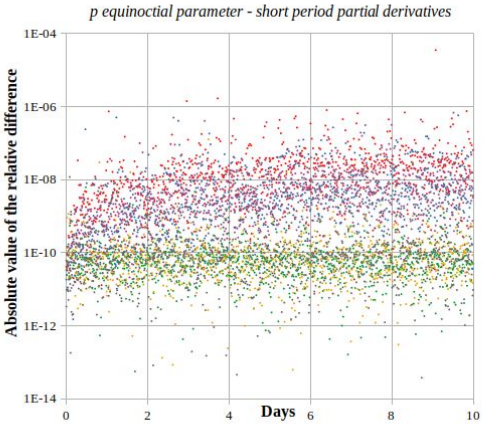
Partial derivatives of h short periodic term

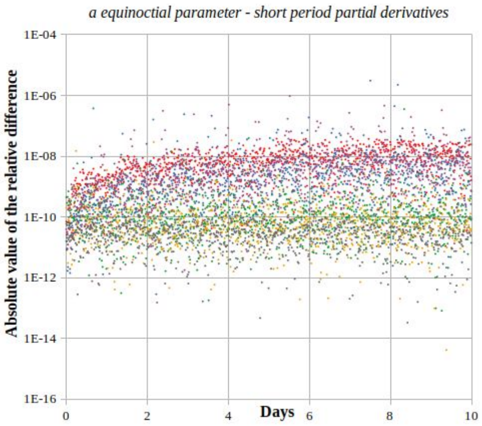
- with respect to $\bar{a}$
- with respect to $\bar{h}$
- with respect to $\bar{k}$
- with respect to $\bar{p}$
- with respect to $\bar{q}$
- with respect to $\bar{\lambda}$

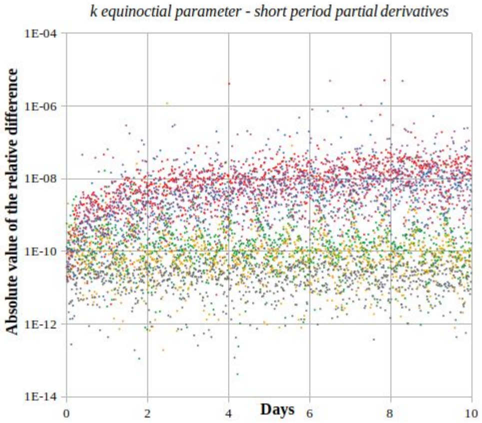
$$B_1 = \left[\frac{\partial \boldsymbol{\varepsilon\eta}(\boldsymbol{a})}{\partial \overline{\boldsymbol{a}}(t)}\right]_{6\times 6}$$

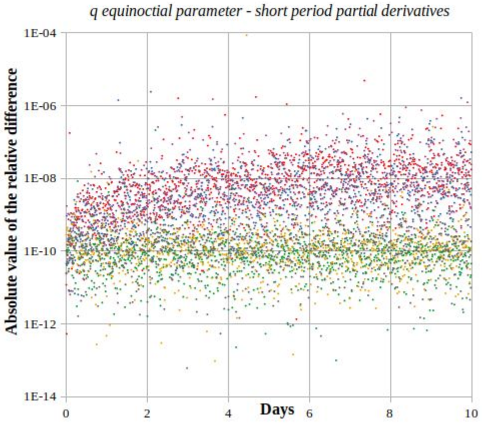
Validation of the State Transition Matrix

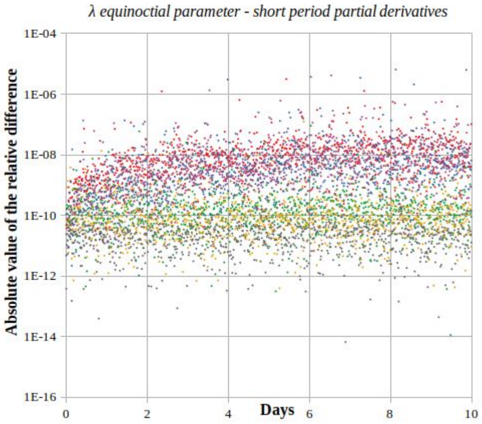






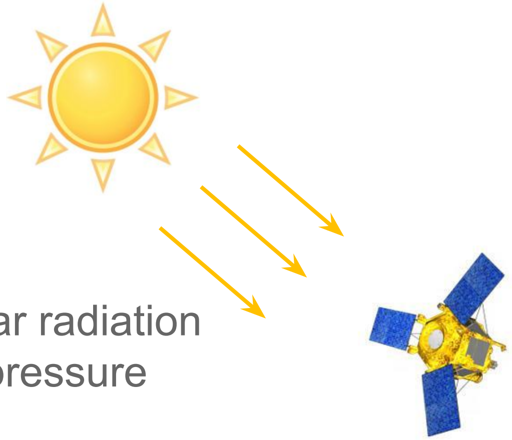






The Batch Least Squares Orbit Determination

Third body attraction



Solar radiation pressure

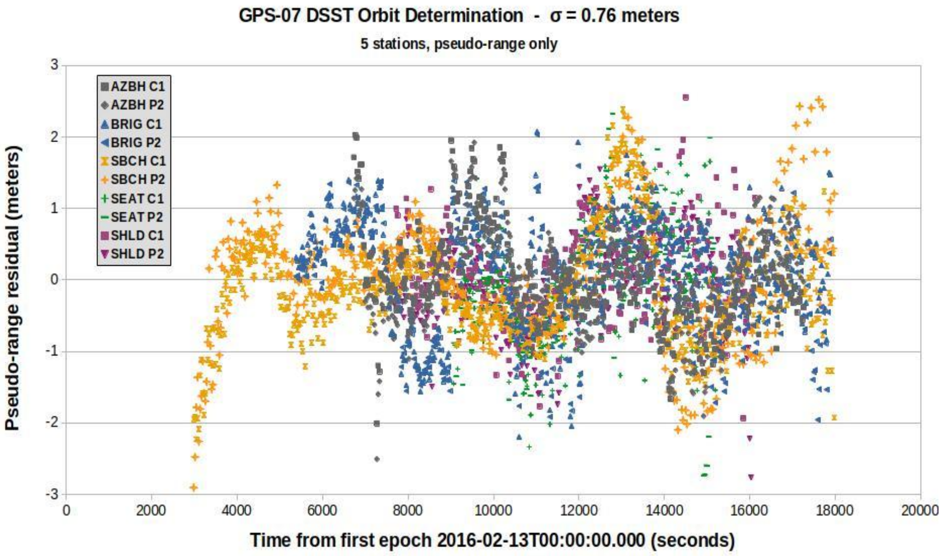
12 x 12 geopotential terms

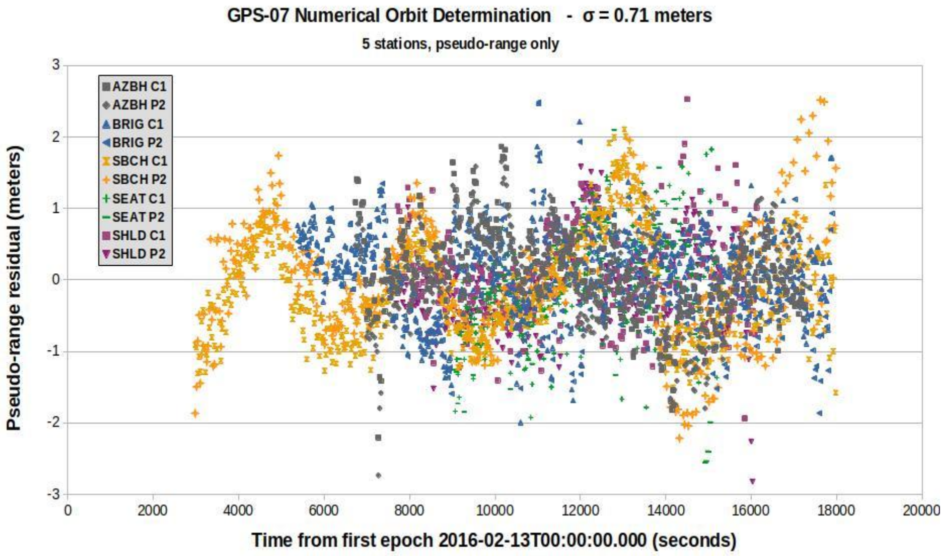
Third body attraction

- 5 stations
- 4009 pseudo-range measurements from Rinex file

The Batch Least Squares Orbit Determination

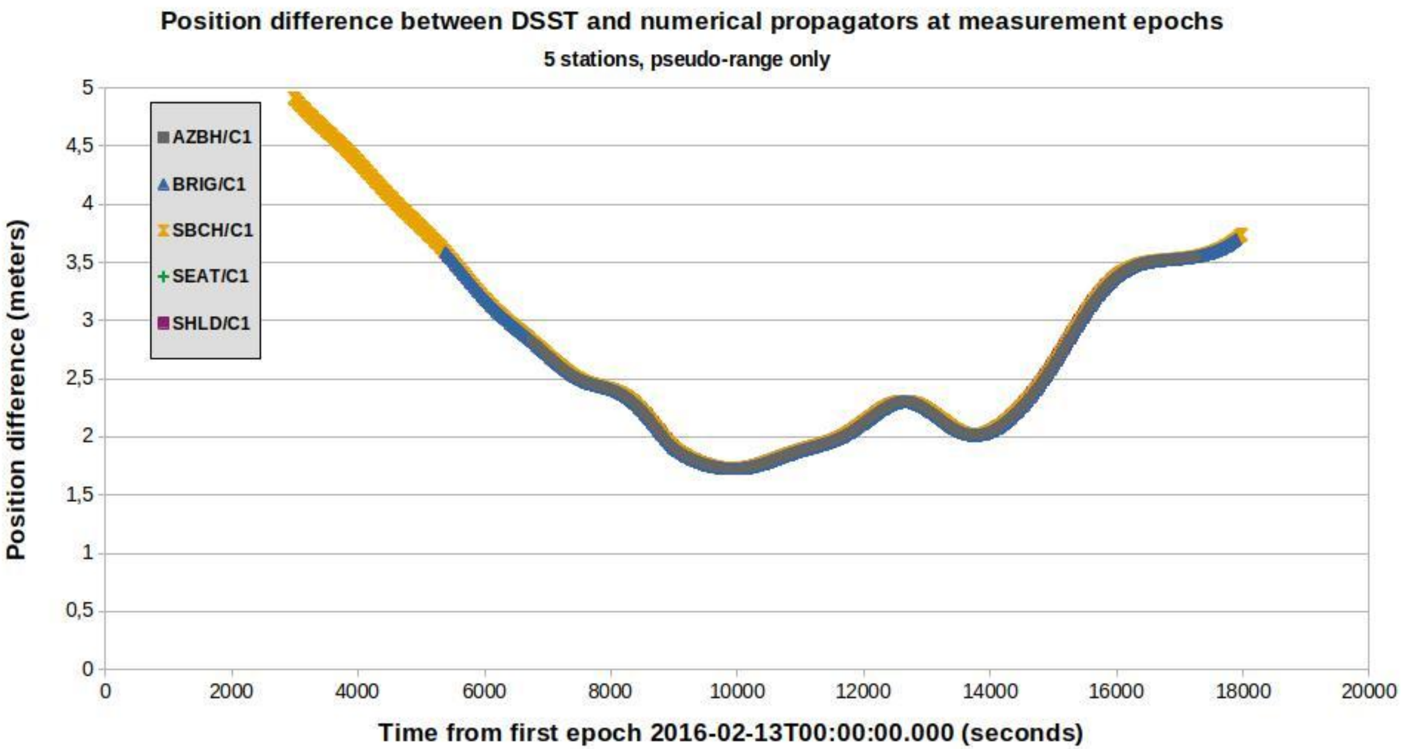
Tutorial: https://gitlab.orekit.org/orekit/orekit-tutorials/-/blob/master/src/main/java/org/orekit/tutorials/estimation/DSSTOrbitDetermination.java





(Cazabonne and Cefola, AAS Paper 21-309)

The Batch Least Squares Orbit Determination



The Extended Semi-analytical Kalman Filter (ESKF)

Orekit Talk - 23th of May 2023

Objective of the ESKF

Main issue

- EKF needs to be re-initialized at each measurement epoch
- DSST uses very large step size to integrate mean elements (e.g., half a day)

Goal

Reconcile the conflicting goal of the DSST perturbation theory and the EKF theory

- Avoid numerical re-initialization of the nominal state at each measurement epoch
- Distinction between operations in integration and observations grid

➡ The **Extended Semi-analytical Kalman Filter** **(Taylor, 1981)** **(Wagner, 1983)** **(Folcik, 2008)**

Algorithm of the ESKF

See Appendix D for the workflow

Initialization of the algorithm

$\Delta\widehat{\boldsymbol{Y}}_0 = 0$ $\qquad \phi_{t_0,t_0} = I$ $\qquad \Psi_{t_0,t_0} = 0$

Operations on the Integration Grid

Update nominal state $\boldsymbol{Y}_{\boldsymbol{k}}$

Integrate to obtain $\overline{\boldsymbol{a}}_N(t_k)$, Φ_{t_k,t_0}, and Ψ_{t_k,t_0}

Calculate short periodics $\varepsilon C_\sigma(\overline{\boldsymbol{a}}_N)$ and $\varepsilon D_\sigma(\overline{\boldsymbol{a}}_N)$

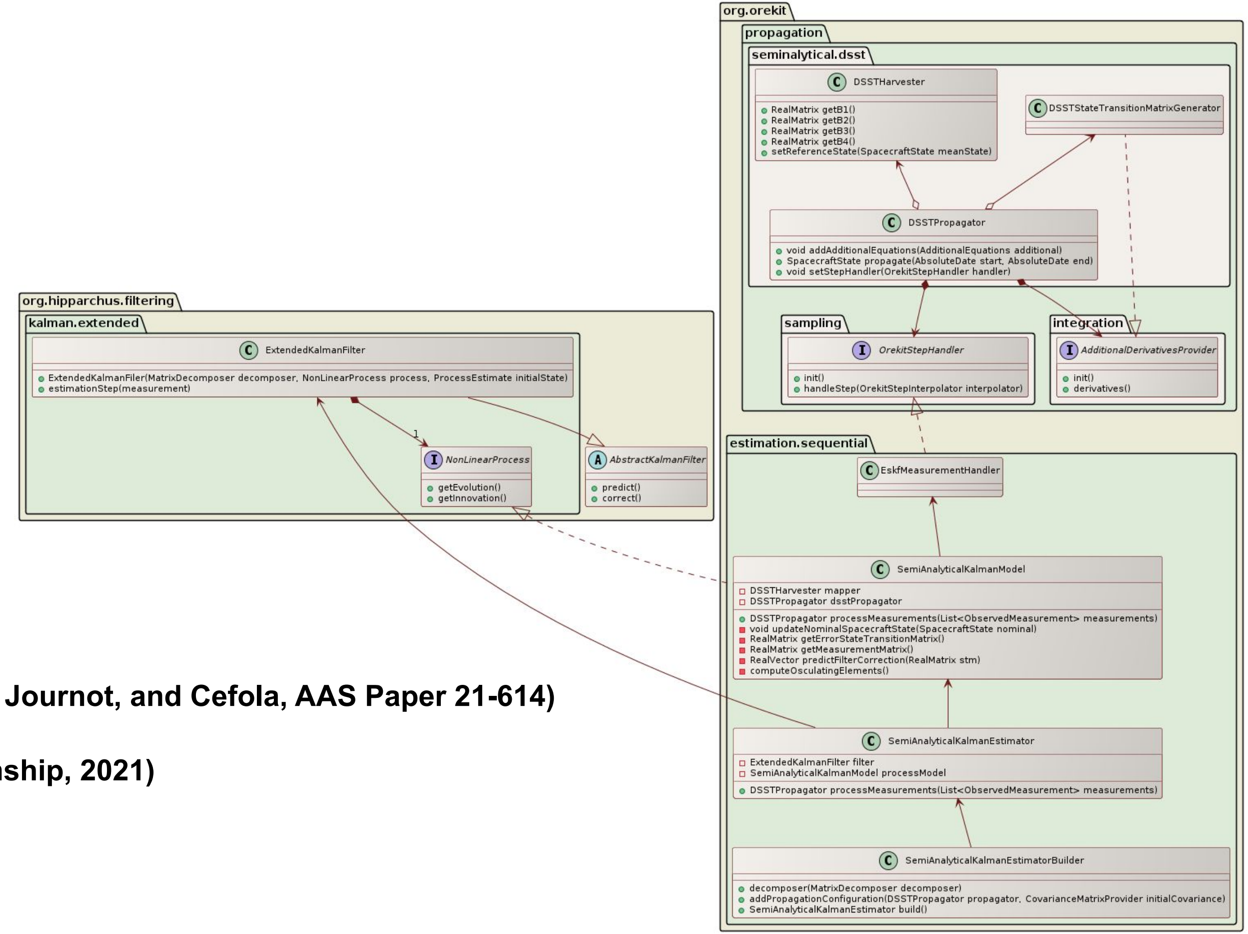
Operations on the Observations Grid

Interpolate to obtain $\overline{\boldsymbol{a}}_N(t_k)$, Φ_{t_k,t_0}, and Ψ_{t_k,t_0}

Calculate $\Delta\widehat{\boldsymbol{Y}}_k$

Calculate $\widehat{P}_k$

ESKF architecture in Orekit



(Cazabonne, Bayard, Journot, and Cefola, AAS Paper 21-614)

(Julie Bayard’s internship, 2021)

ESKF validation against simulated measurements

3 test cases for the validation

Two bodies attraction

Case 1

Two bodies attraction $+ J_{2,0}$

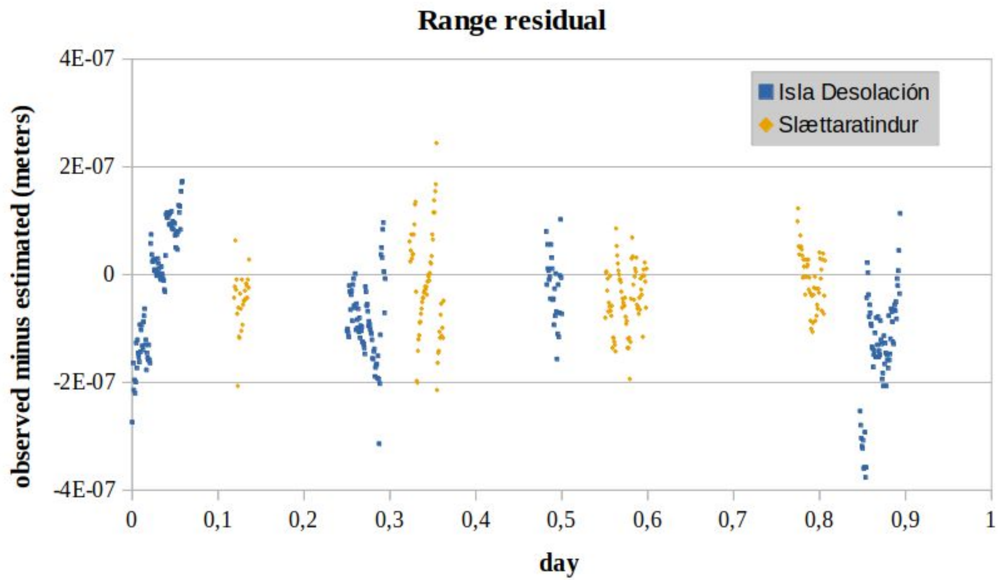
Case 2

Two bodies attraction $+ J_{2,0} + J_{2,1} + J_{2,2}$

Case 3

- 2 stations
- 445 simulated pseudo-range measurements

ESKF validation against simulated measurements

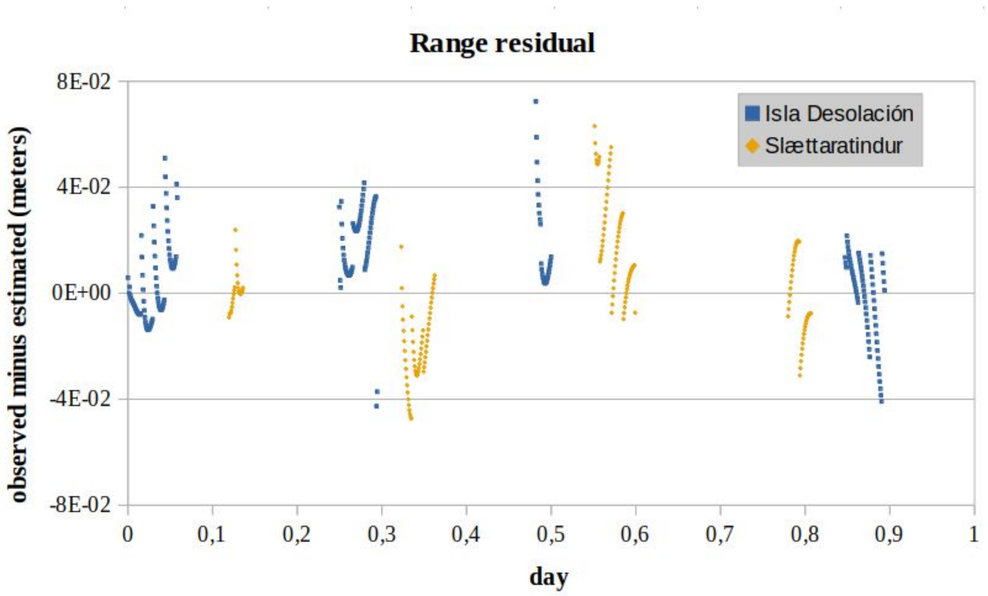


Case 1

Mean residuals value: 4.97E-08 meters

Perfect estimation of the last epoch orbit!

ESKF validation against simulated measurements



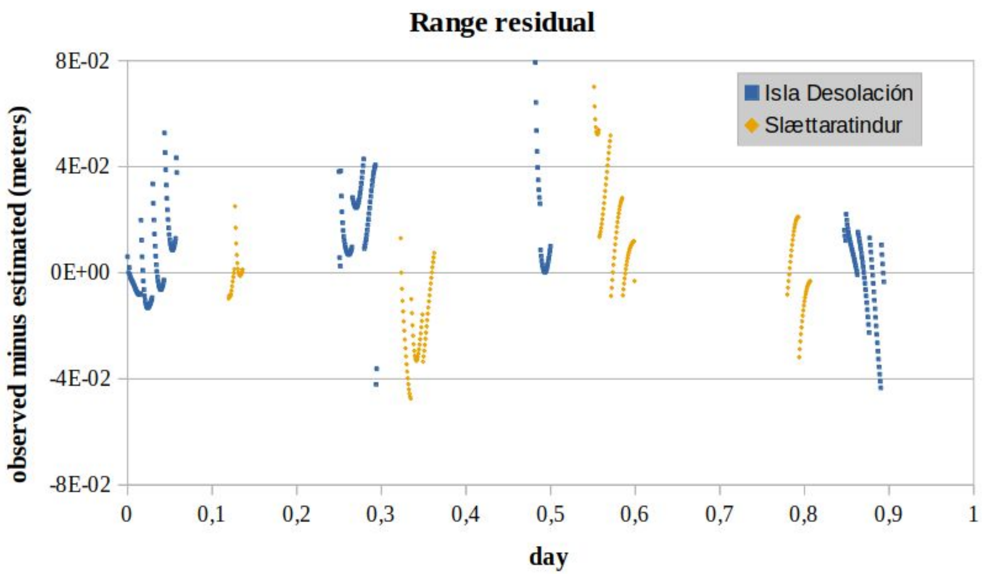
Case 2

Mean residuals value: 3.88E-03 meters

Initial position error: 1.05 meters

Final position error: 4.5 centimeters

ESKF validation against simulated measurements



Case 3

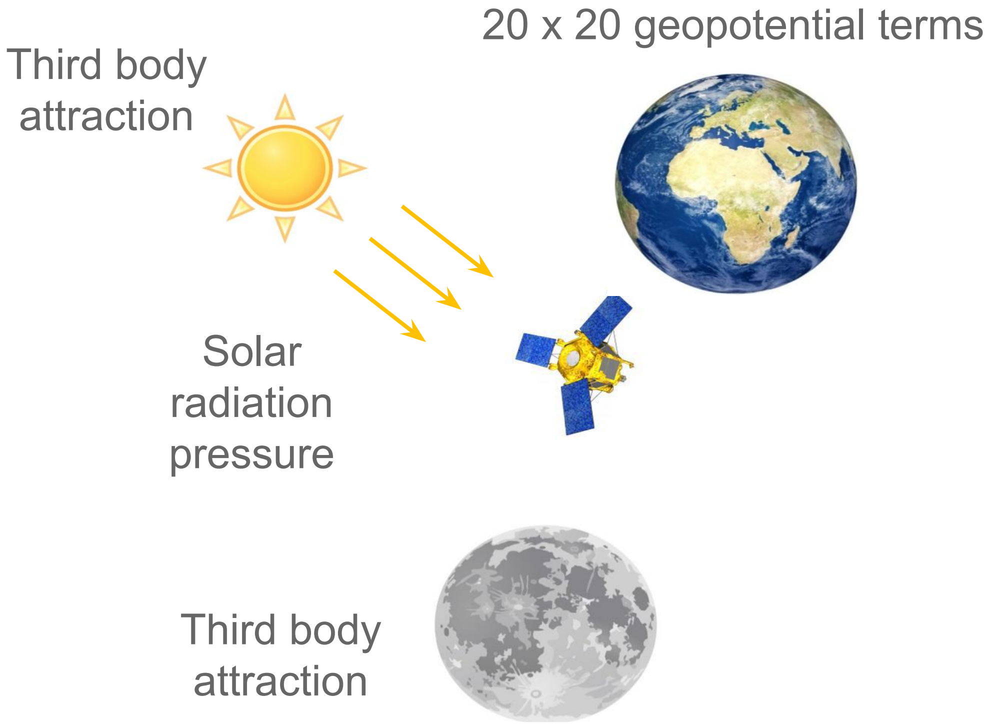
Mean residuals value: 3.94E-03 meters

Initial position error: 1.05 meters

Final position error: 4.1 centimeters

ESKF Validation against real data

Validation against real data (CDDIS): Lageos 2 satellite

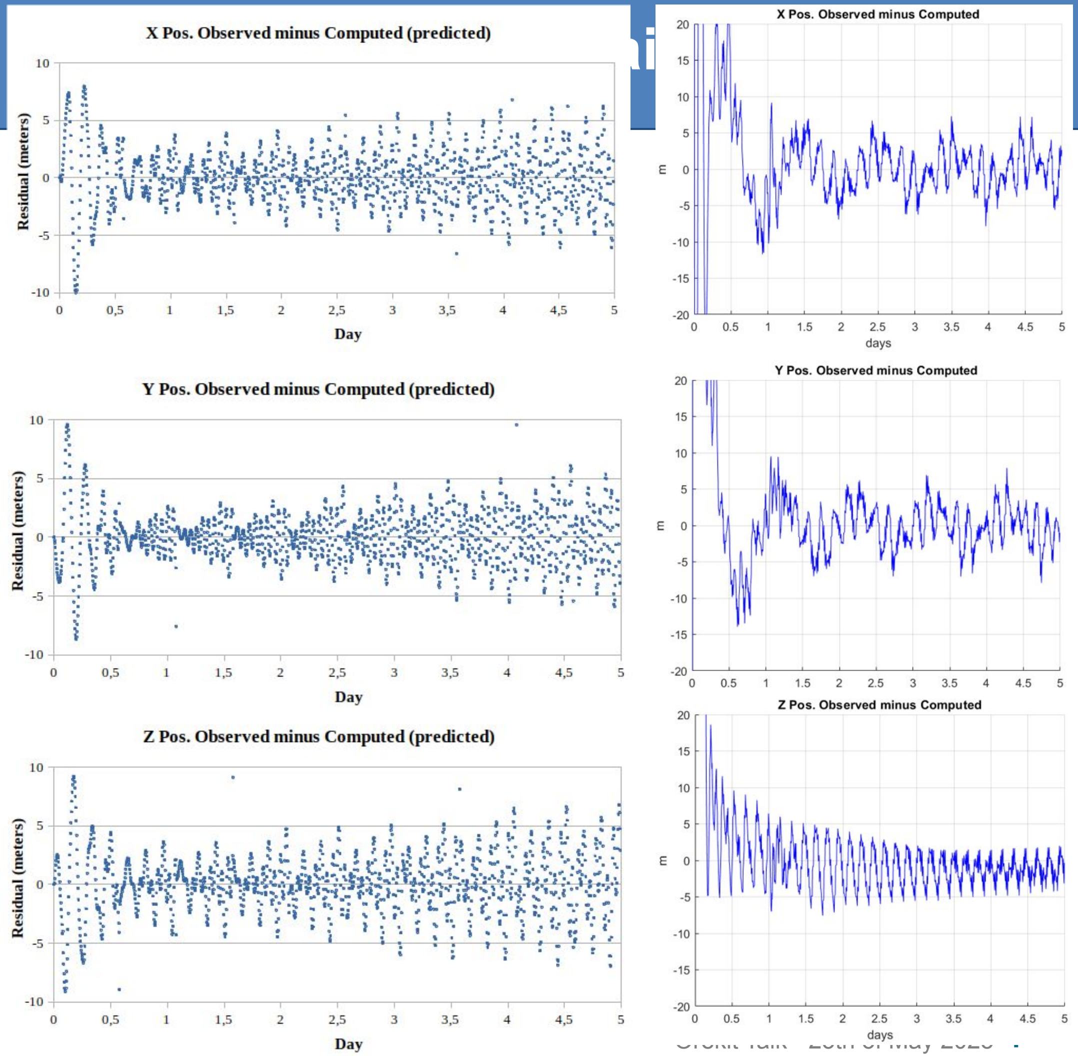


Goal

- Compare the results with the GTDS ESKF to validate the residuals

Test case

- An observation each 5 minutes
- 1220 position measurements (CPF) on 5 days



Comparison with GTDS ESKF

Same order of magnitude as GTDS for measurement residuals

⇒Small increase of the residuals over time in Orekit DSST

How improving ESKF results?

The corrected measurement residual

(Cazabonne, Bayard, Journot, and Cefola, AAS Paper 21-614)

- No present in the previous reference ESKF papers - Orekit contribution

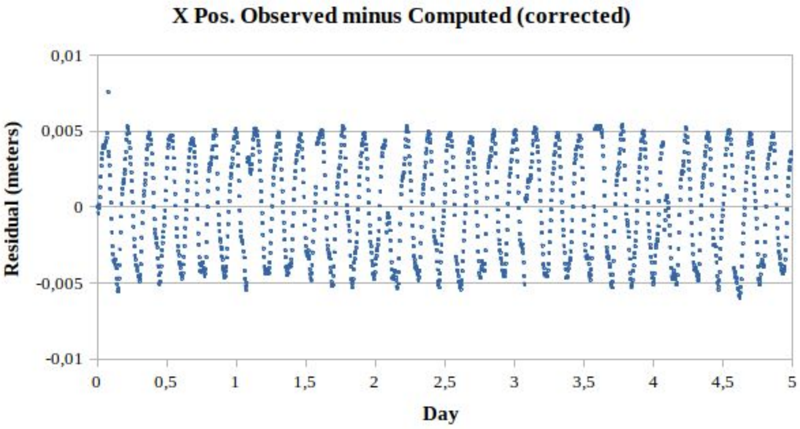
11. Calculate the corrected measurement and residual using the corrected osculating elements given in Equation (27).

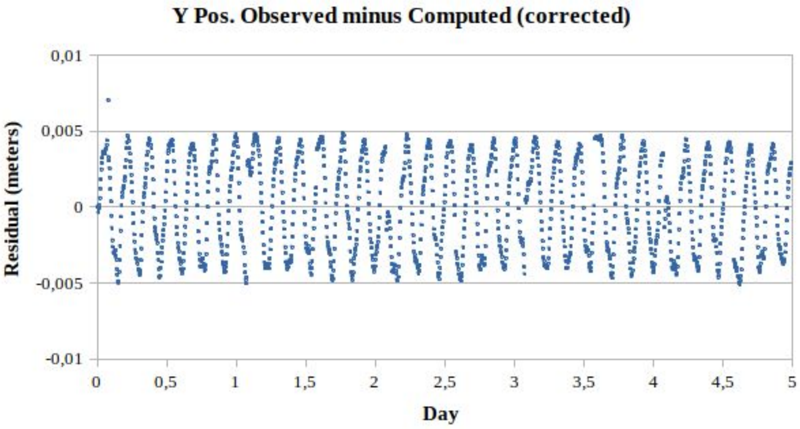
$$\hat{\boldsymbol{a}}^*(t_k) = \bar{\boldsymbol{a}}_N(t_k) + \Delta\hat{\boldsymbol{a}}_k + \varepsilon\eta(\bar{\boldsymbol{a}}_N) + B_1\Delta\hat{\boldsymbol{a}}_k \quad (27)$$

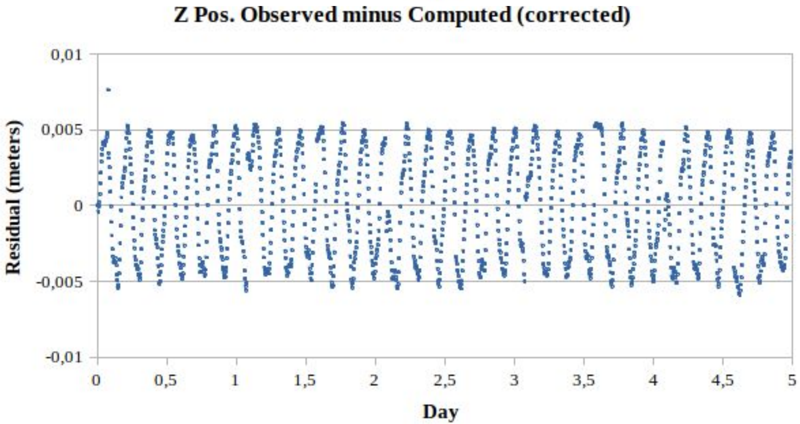
→ Significant impact on the previous results

ESKF Validation against real data

Tutorial:
https://gitlab.orekit.org/orekit/orekit-tutorials/-/blob/master/src/main/java/org/orekit/tutorials/estimation/ExtendedSemianalyticalKalmanFilter.java







30% faster than a numerical estimator and precision is almost the same !

The Unscented Semi-analytical Kalman Filter (USKF)

Orekit Talk - 23th of May 2023

Algorithm of the USKF

See Appendix E for the workflow

Initialization of the algorithm

$\Delta\widehat{\boldsymbol{Y}}_0 = 0$

P_0

$\widehat{\boldsymbol{Y}}_{\mathbf{0}}$

Operations on the Integration Grid

Update nominal state $\boldsymbol{Y}_{\boldsymbol{k}}$

Integrate to obtain $\overline{\boldsymbol{a}}_N(t_k)$

Calculate short periodics $\varepsilon C_\sigma(\overline{\boldsymbol{a}}_N)$ and $\varepsilon D_\sigma(\overline{\boldsymbol{a}}_N)$

Operations on the Observations Grid

Interpolate to obtain $\overline{\boldsymbol{a}}_N(t_k)$

Calculate the sigma points $\Delta\boldsymbol{Y}_{k-1}^i$

Calculate $\widehat{P}_k$

Algorithm of the USKF

Sigma points of the filter correction

$$\Delta \boldsymbol{Y}^i_{k-1} = \begin{cases} \Delta \widehat{\boldsymbol{Y}}_{k-1} + \left(\sqrt{(n+\lambda)\widehat{P}_{k-1}}\right)_i & i = 1, \dots, n \\ \Delta \widehat{\boldsymbol{Y}}_{k-1} & i = 0 \\ \Delta \widehat{\boldsymbol{Y}}_{k-1} - \left(\sqrt{(n+\lambda)\widehat{P}_{k-1}}\right)_i & i = n+1, \dots, 2n \end{cases}$$

Predicted sigma points

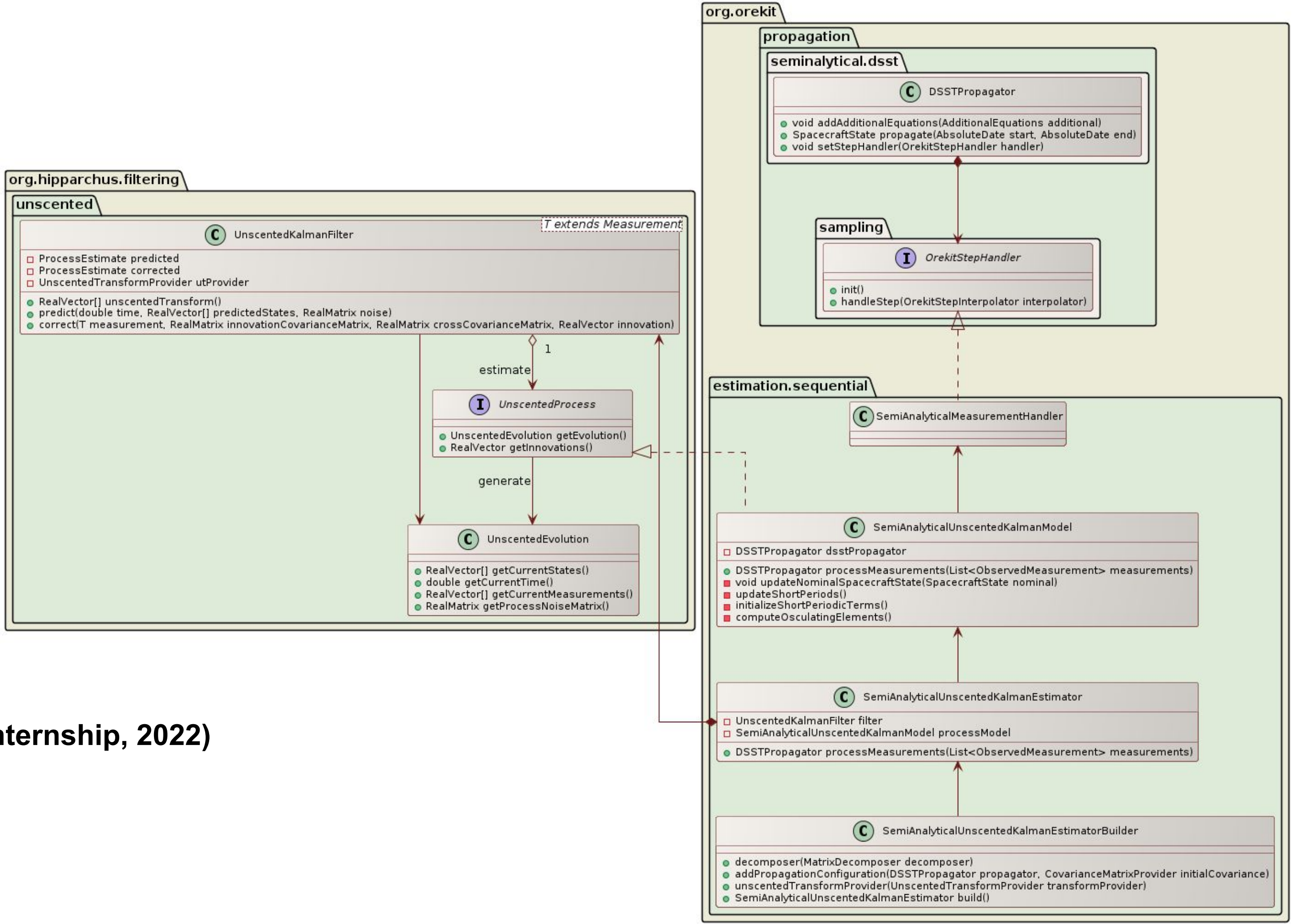
$$\Delta \overline{\boldsymbol{Y}}_k = \begin{bmatrix} \Delta \overline{\boldsymbol{\alpha}}_k \\ \Delta \overline{\boldsymbol{\varsigma}}_k \end{bmatrix}$$

$$\Delta \overline{\boldsymbol{\alpha}}_k = \Phi_{t_k, t_{k-1}} \Delta \widehat{\boldsymbol{\alpha}}_{k-1}$$

$$\Delta \overline{\boldsymbol{\varsigma}}_k = \Delta \widehat{\boldsymbol{\varsigma}}_{k-1}$$

$$\Phi_{t_k, t_{k-1}} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \dfrac{\partial \bar{\lambda}_{t_k}}{\partial \bar{a}_{1_{t_{k-1}}}} & \cdots & 1 \end{pmatrix}$$

USKF architecture in Orekit

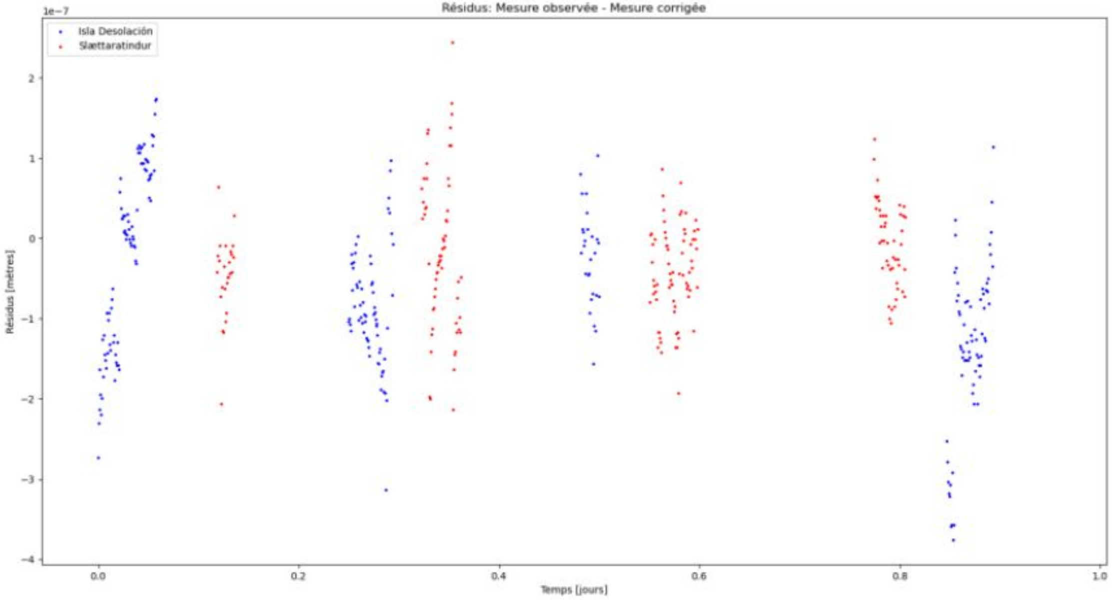


(Gaëtan Pierre’s internship, 2022)

USKF validation against simulated measurements

Keplerian motion test cases for the validation

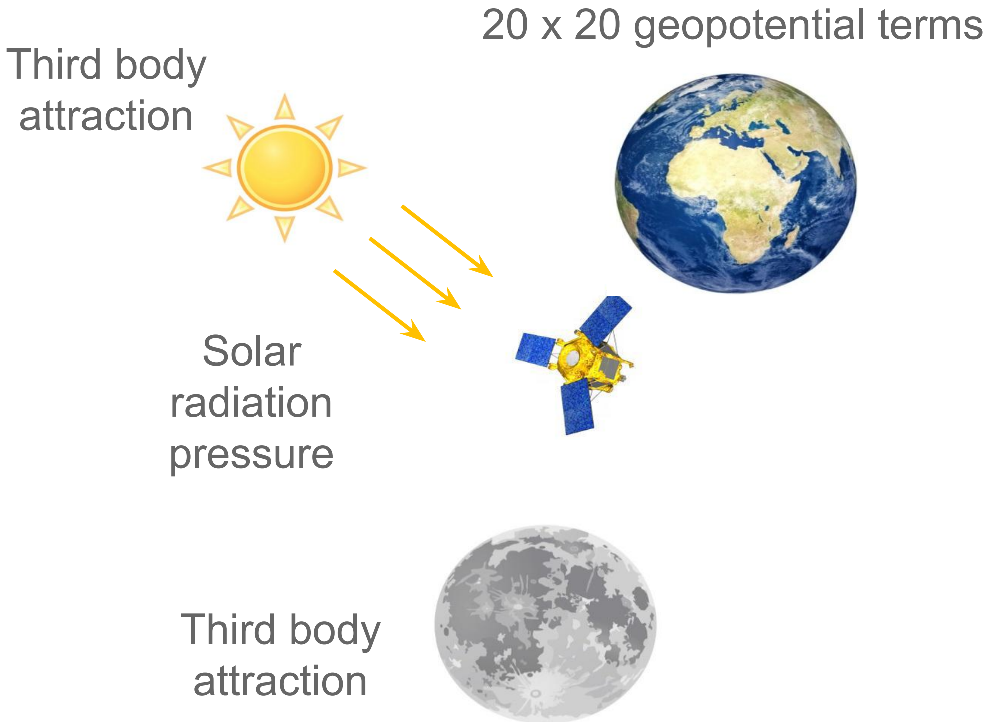
Two bodies attraction



- 2 stations
- 445 simulated pseudo-range measurements

USKF Validation against real data

Validation against real data (CDDIS): Lageos 2 satellite



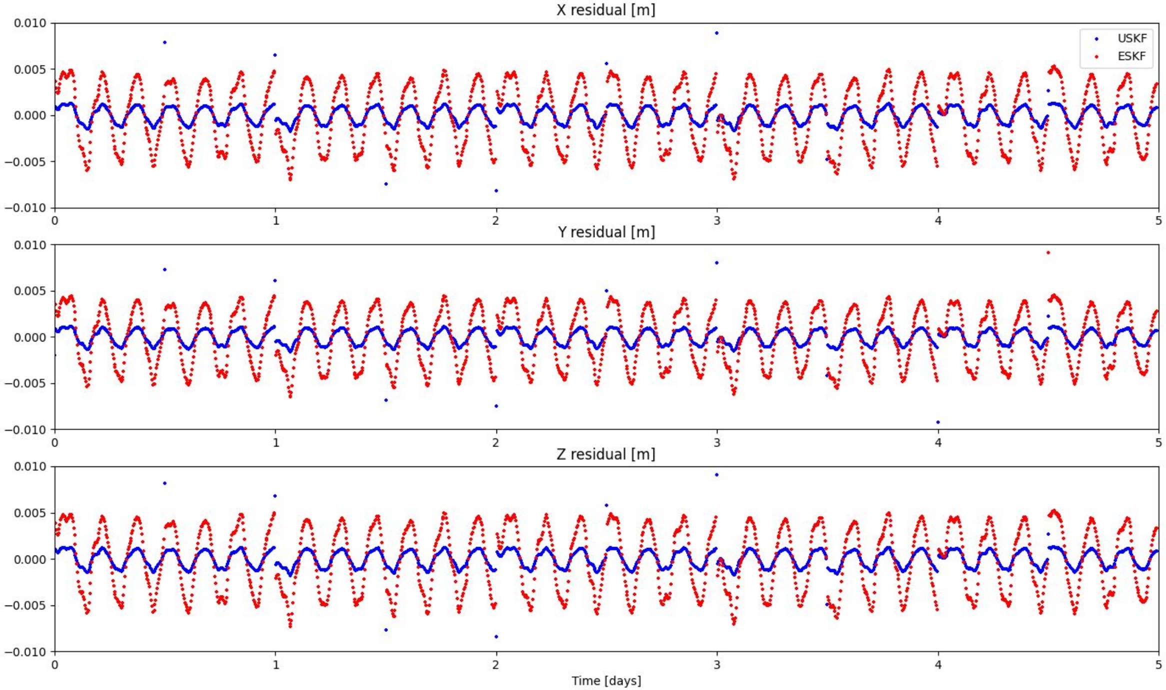
Goal

- Compare USKF results with the already validated ESKF

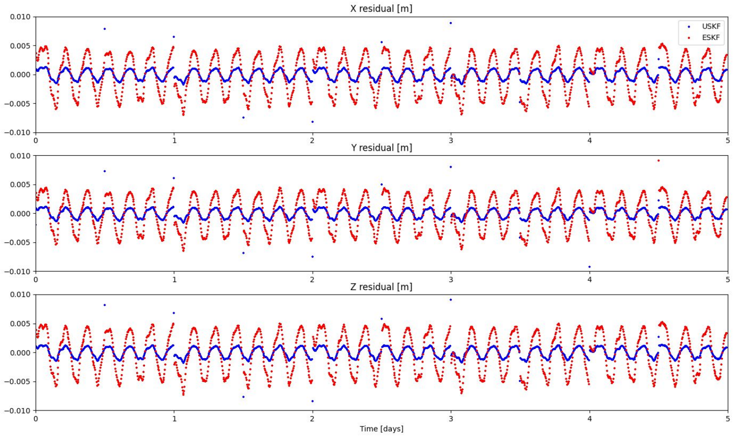
Test case

- An observation each 5 minutes
- 1220 position measurements (CPF) on 5 days

USKF Validation against real data



USKF Validation against real data



Interpretation

- USKF better accuracy is anecdotal. Results are on the same order of magnitudes!

➡ USKF first version gives promising results

	ESKF			USKF		
	X	Y	Z	X	Y	Z
Mean residual (m)	-1.45e-4	-1.30e-4	-1.48e-4	-2.84e-5	-3.31e-5	-8.55e-5
Standard Deviation (m)	4.14e-3	3.72e-3	4.23e-3	1.06e-3	0.92e-3	2.14e-3

Conclusion and Future work

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Conclusion - Orekit and DSST during the last 5 years

3 semi-analytical estimation algorithms implemented in Orekit

- 1 Batch Least Squares and 2 Recursive Filters : ESKF and USKF

3 internships

- B. Cazabonne, Orbit Determination with Semi-analytical Theory, Université Paul Sabatier, 2018.
- J. Bayard, Estimation Algorithms for Orbit Determination, ISAE Supaero, 2021.
- G. Pierre, Orbit Determination with Unscented Kalman Filters, ISAE Supaero, 2022.

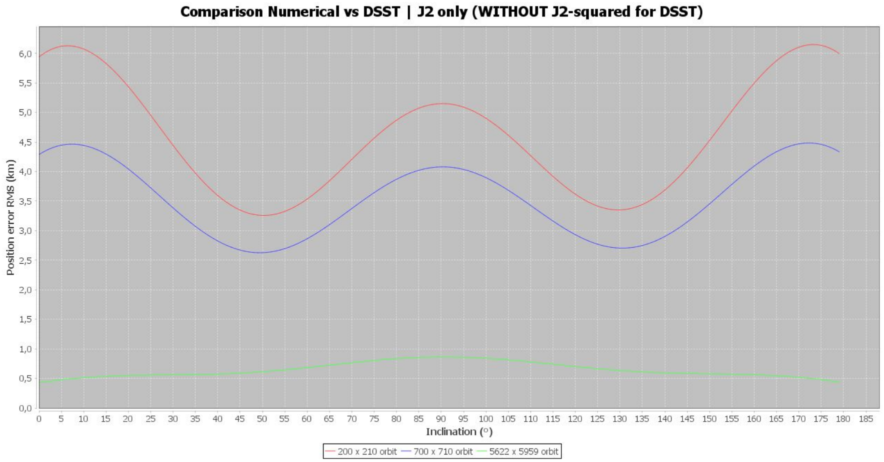
3 conference papers

- Cazabonne B., and Maisonobe L., Open source orbit determination using semi-analytical theory, 7th ICATT, Oberpfaffenhofen, Germany, November 6-9, 2018.
- Cazabonne B., and Cefola P. J., Towards Accurate Orbit Determination using Semi-analytical Satellite Theory, AAS Paper 21-309, 31st AAS/AIAA Space Flight Mechanics Meeting, Virtual, February 1-4, 2021.
- Cazabonne B., Bayard J., Journot M., Cefola P. J., A Semi-analytical Approach for Orbit Determination based on Extended Kalman Filter, AAS Paper 21-614, AAS/AIAA Astrodynamics Specialist Conference, Big Sky, Virtual, August 9-11, 2021.

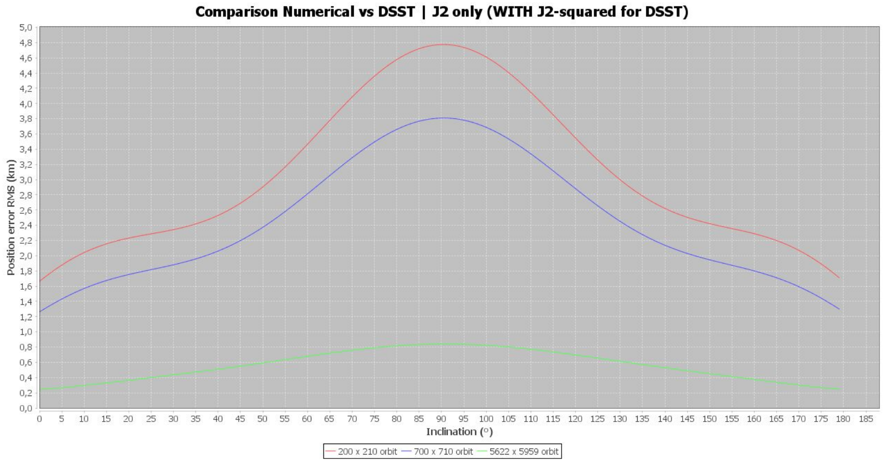
Future work - Improve dynamical models

J2-squared effect - Zeis Model (Orekit 12.0)

Without J2-squared



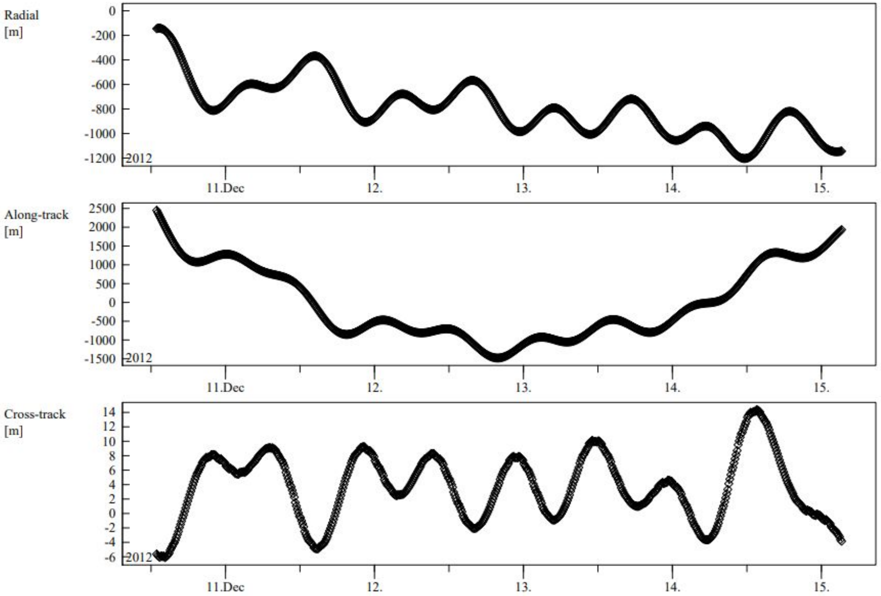
With J2-squared



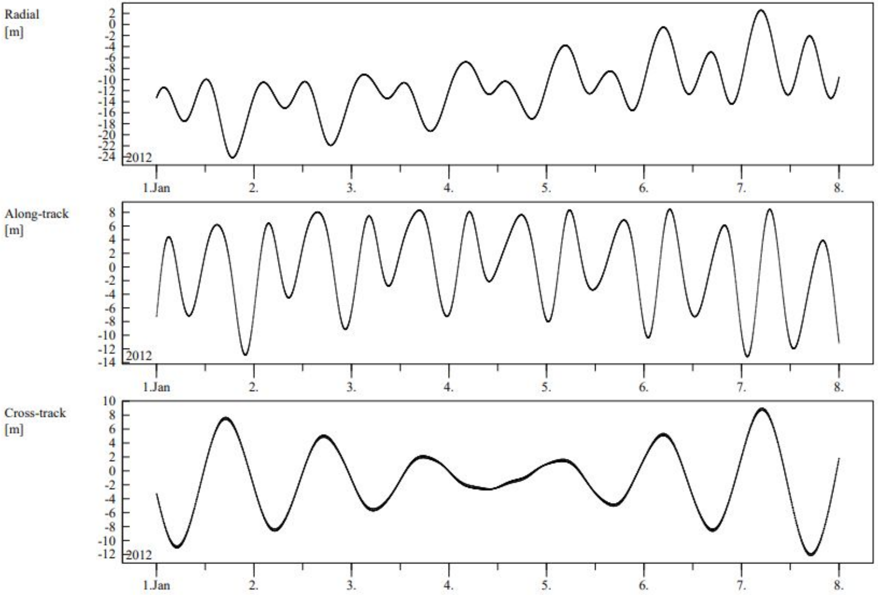
Future work - Improve dynamical models

Weak time dependant terms closed form (Orekit ?)

Without WTD terms



With WTD terms



(S. Setty PHD thesis)

Future work

More validation

- Validation of ESKF and USKF against station measurements (e.g., Rinex, SLR, etc.)
- Validation of ESKF and USKF against different orbit types (e.g. LEO, GEO)

→ **CDDIS**

Multi-satellites orbit determination

- Extend ESKF and USKF algorithms to perform multi-satellites orbit determination
- Validation examples: GRACE, GNSS constellation (.sp3 files)

Fixing bugs

Write a paper for USKF algorithm

Thank you!

Particular thanks to: Luc Maisonobe, Paul Cefola, Maxime Journot, Pascal Parraud, Julie Bayard, Gaëtan Pierre

Orekit Talk - 23th of May 2023

Appendix A - Variational Equations

Variational equations

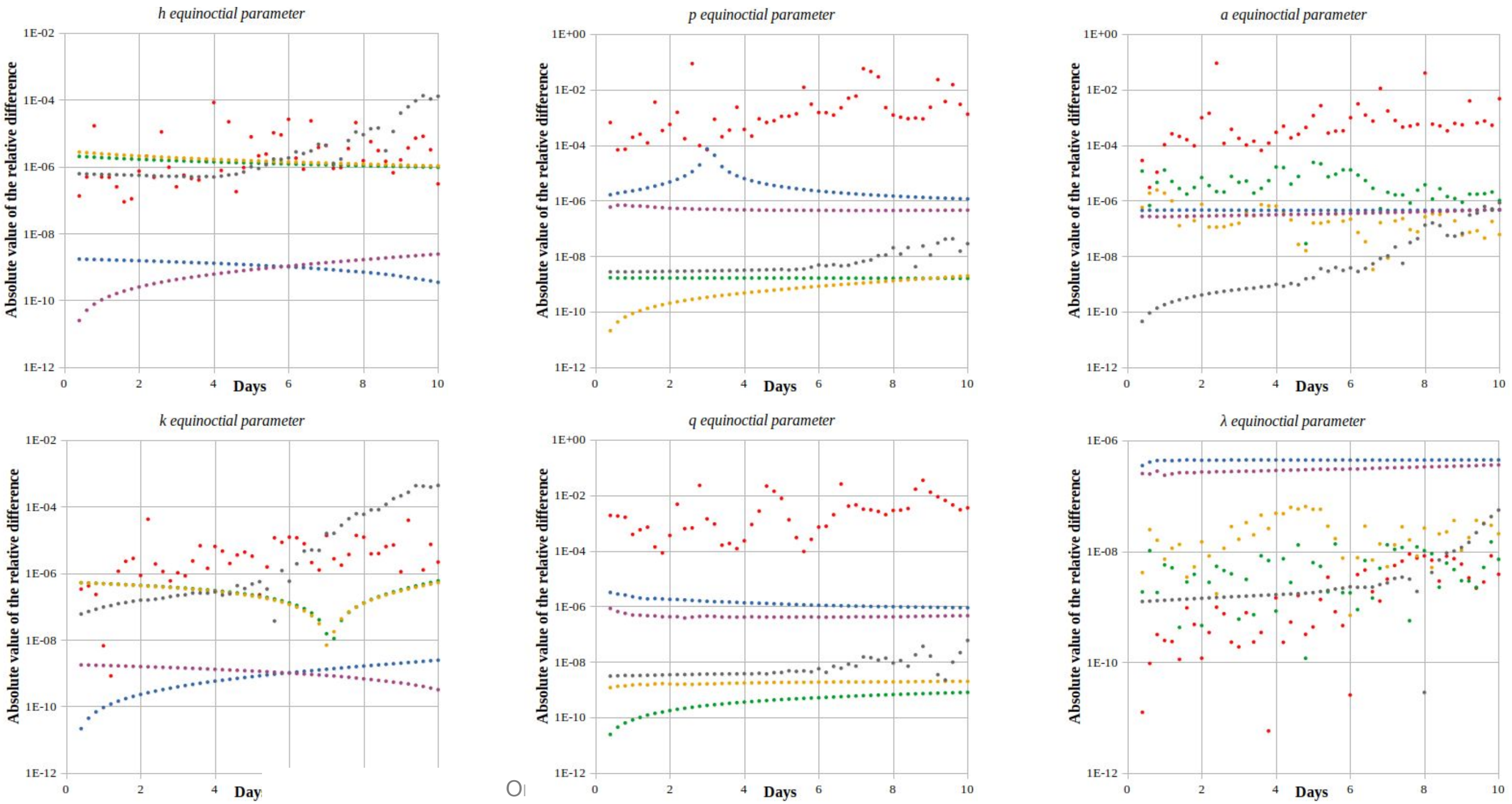
$$\frac{d}{dt}B_2 = K.B_2 \quad \textit{with } [B_2]_{t_0} = I_6$$

[illegible] = [illegible] 6×6

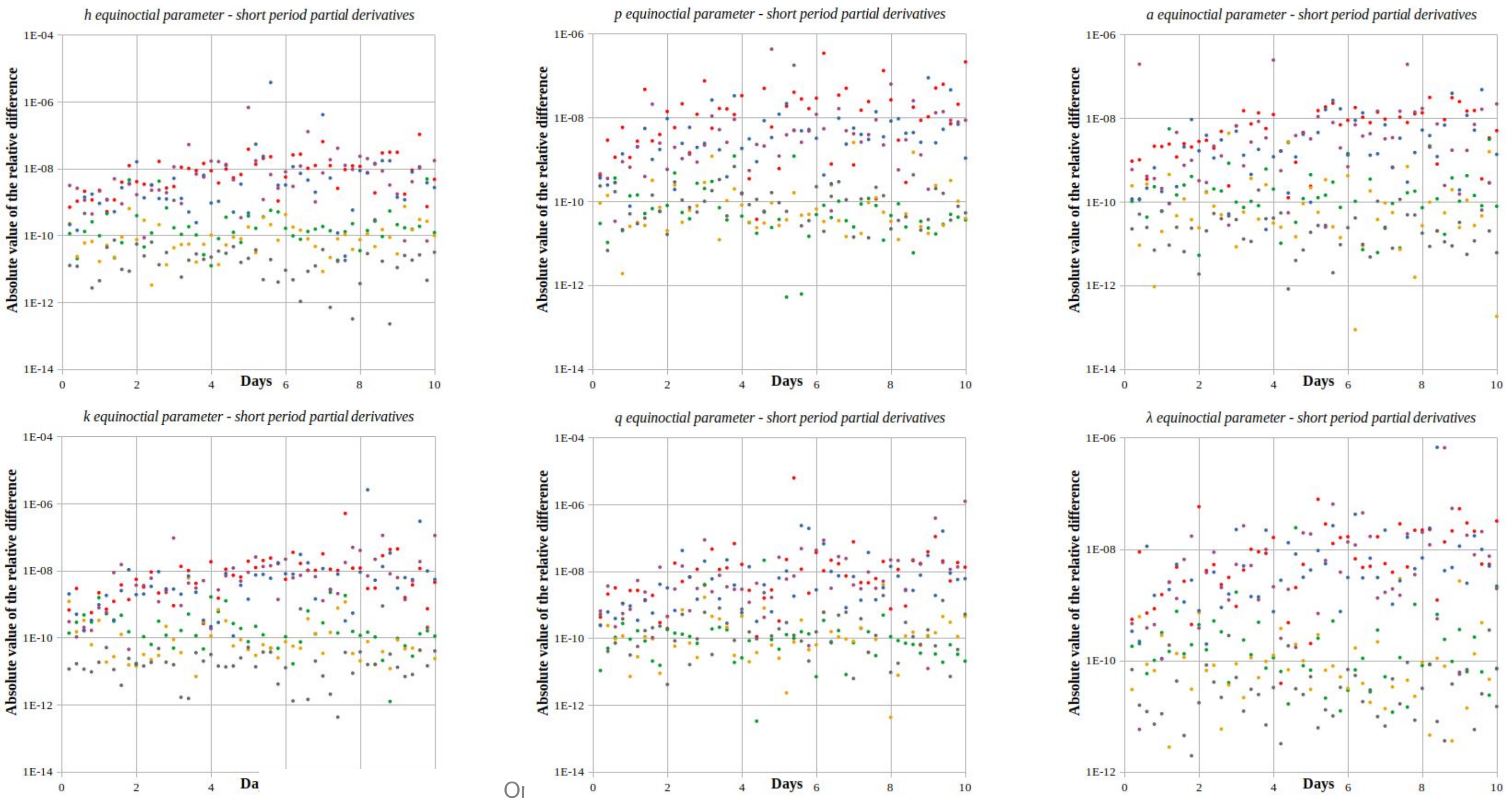
$$\frac{d}{dt}B_3 = K.B_3 + C \quad \textit{with } [B_3]_{t_0} = 0_{6\times(s-6)}$$

[illegible] = [illegible] $6\times(\,[illegible] - 6)$

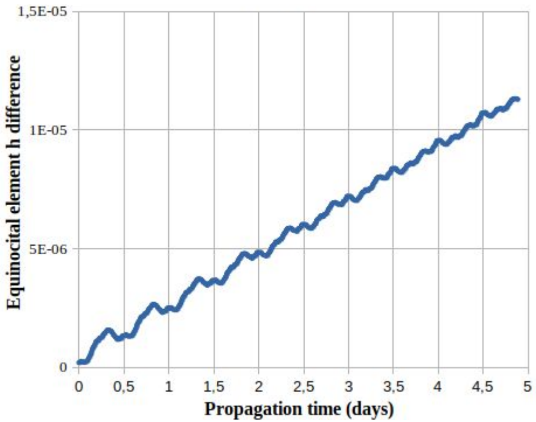
Appendix B.1 - STM validation including drag effect

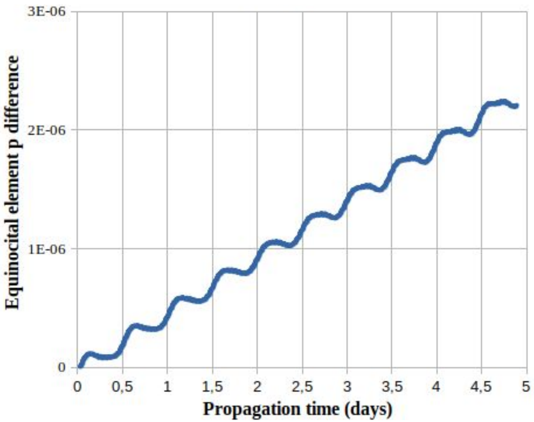


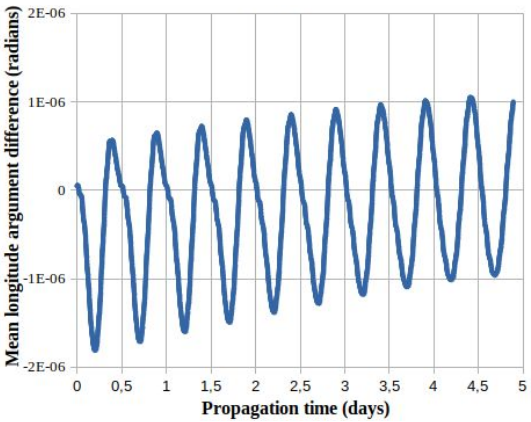
Appendix B.2 - STM validation including drag effect

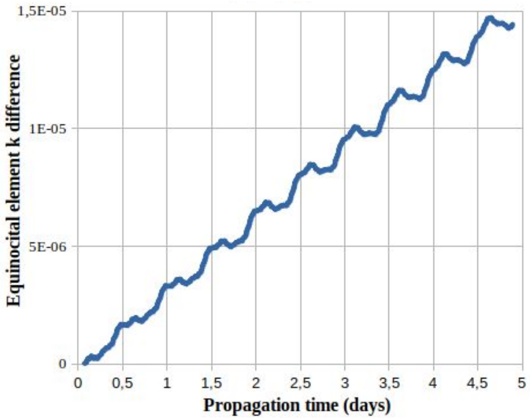


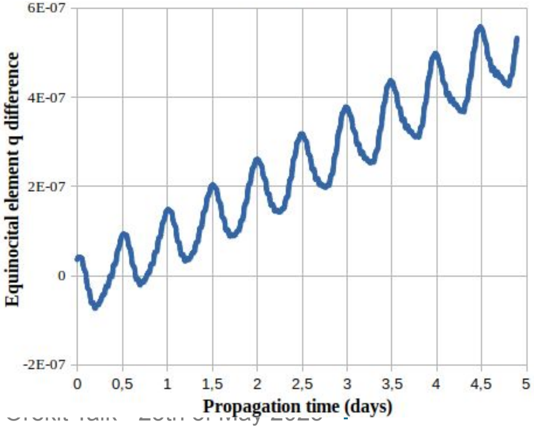
Appendix C.1 - 5 days propagation of estimated (LS) GPS orbit

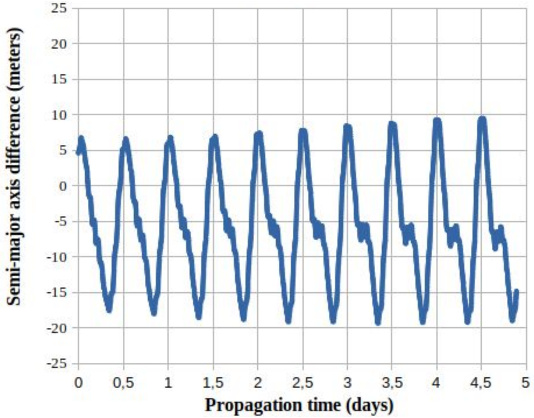




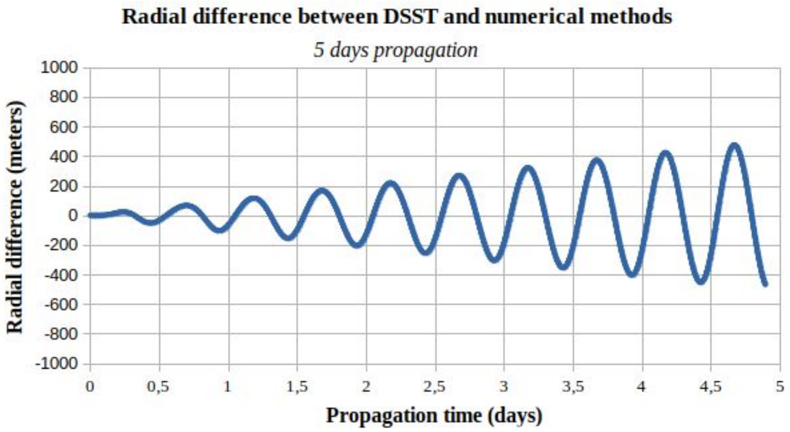


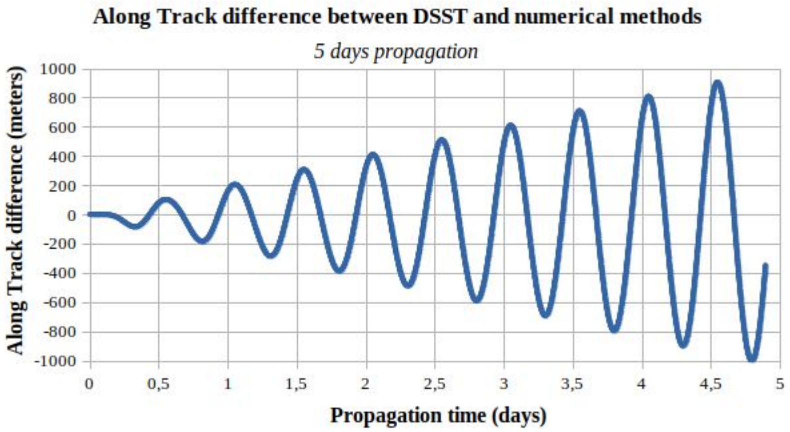


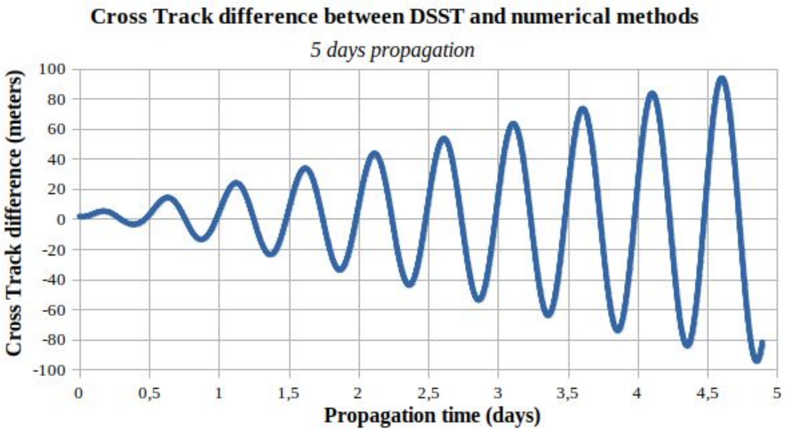




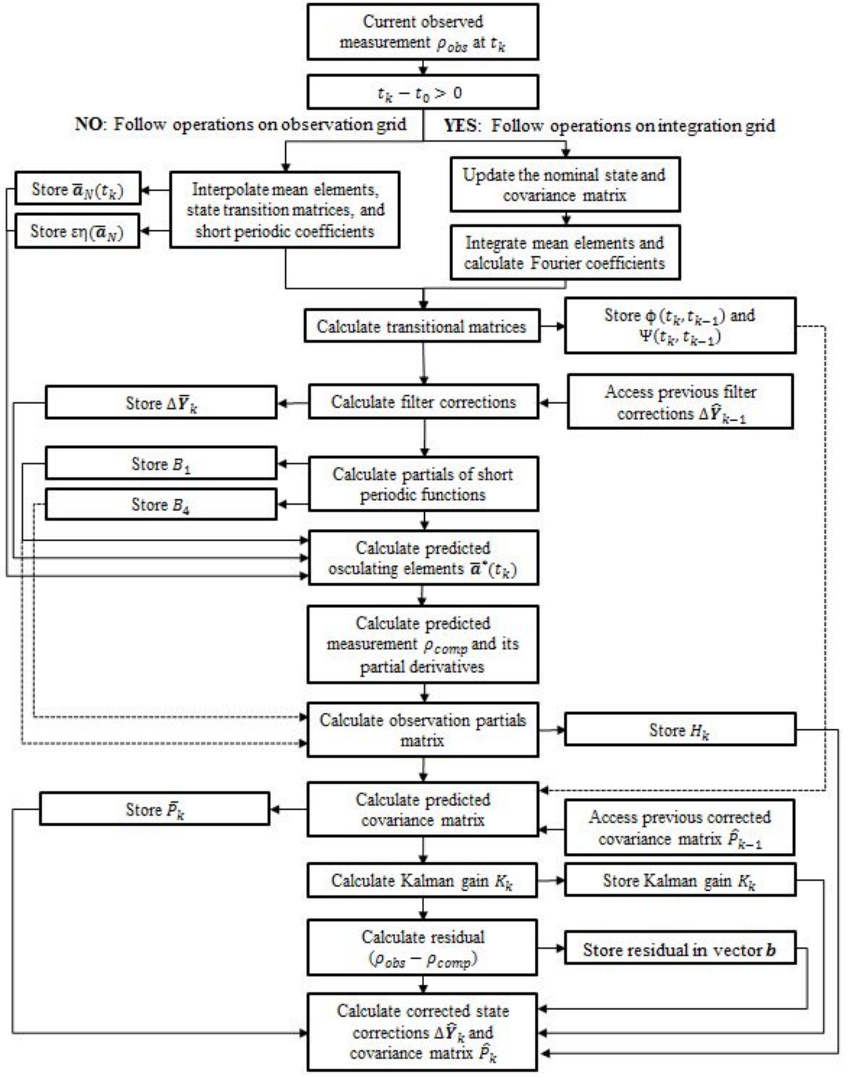
Appendix C.2 - 5 days propagation of estimated (LS) GPS orbit







Appendix D - ESKF A



Appendix E - USKF